ABOUT ASYMPTOTICAL BEHAVIOR OF THE NEUTRONS PHASE DENSITY IN THE CASE OF ISOTROPIC POINT SOURCE

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Abstract

In this paper we are interested in the asymptote of the phase density of the neutrons emitted from single power source which is disposed on the plane and radiating neutrons to the direction $\mu = \mu_0$ with length of the wave $\lambda = \lambda_0$. To this end is used the method of expansions by singular eigenfunctions of the corresponding characteristic equation for the solution of the equation of linear transport theory, which describes penetration of radiation through metals.

Key words and phrases: Transport theory, singular eigenfunctions, boundary value problem.

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1 Introduction

The aim of this paper is to establish the asymptote of the solutions describing penetration of the neutrons through infinite homogeneous isotropic media while considering a plane geometry with mono directional monochromatic source represented by Dirac's δ functions.

Let $G(x_0, \mu_0, \lambda_0; x, \mu, \lambda)$ be the flux of photons or neutrons at the point $x = x_0$; travelling in the direction $\mu = \mu_0$ with an energy represented by suitable parameter λ_0 (wavelength for photons or lethargy for neutrons). This obeys the transport equation [1]

$$\mu \frac{\partial G(x_0, \mu_0, \lambda_0; x, \mu, \lambda)}{\partial x} + G(x_0, \mu_0, \lambda_0; x, \mu, \lambda) \tag{1}$$

$$= \int_a^b \int_{-1}^{+1} k(\mu, \lambda; \mu', \lambda') G(x_0, \mu_0, \lambda_0; x, \mu', \lambda') d\mu' d\lambda' + S(x_0, \mu_0, \lambda_0; x, \mu, \lambda)$$

$$x \in (-\infty, +\infty), \quad \mu \in (-1, +1), \quad \lambda \in [a, b]$$

where the kernel k is the differential probability of scattering so that

$$k(\mu, \lambda; \mu', \lambda') = \sum_{s=0}^{n} (2s+1) k_s(\lambda, \lambda') p_s(\mu) p_s(\mu'),$$

$p_s(\mu)$ is the Legendre polynomial of order s. S represents the source distribution

$$S(x,\mu,\lambda) = \frac{1}{2\pi}\delta(x-x_0)\delta(\mu-\mu_0)\delta(\lambda-\lambda_0).$$

It is possible to reduce the equation (1) to the homogeneous, if we replace the source by the jumps condition in origin of coordinates. Thus, the equation we must solve then becomes

$$\mu\frac{\partial G}{\partial x} + G = \int_a^b\int_{-1}^{+1}\sum_{s=0}^{n}(2s+1)k_s(\lambda,\lambda')p_s(\mu)p_s(\mu')G d\mu' d\lambda' \tag{2}$$

satisfying the boundary condition

$$2\pi\mu(G(x_0,\mu_0,\lambda_0;x_0^+,\mu,\lambda) - G(x_0,\mu_0,\lambda_0;x_0^-,\mu,\lambda)) = \delta(\mu-\mu_0)\delta(\lambda-\lambda_0) \tag{3}$$

and satisfying also the addition condition on the infinity

$$\lim_{|x|\to\infty} G(x_0,\mu_0,\lambda_0;x,\mu,\lambda) = 0, \tag{4}$$

$$\mu,\mu_0 \in (-1,+1), \quad \lambda,\lambda_0 \in [a,b].$$

2 Singular Eigenfunctions

The translational symmetry of Eq. (2) suggest looking for solutions of the form

$$G(x,\mu,\lambda) = \exp(\frac{x-x_0}{\nu})\varphi_\nu(\mu,\lambda)$$

which gives the following characteristic equation of the transport theory

$$(\nu-\mu)\varphi_\nu(\mu,\lambda) = \nu\int_a^b\int_{-1}^{+1} k(\mu,\lambda;\mu',\lambda')\varphi_\nu(\mu',\lambda')d\mu' d\lambda' \tag{5}$$

where ν is a parameter.

The values of ν for which Eq.(5) has nonzero continuous solution are the discrete eigenvalues of the operator corresponding to Eq. (5). Discrete eigenvalues ν_i are not present for certain kernel of equation. For example, when the integral operator is Volterra operator with respect to variable λ, the discrete eigenvalues ν_i are not present. Further we shall be restricted by this case, (i.e. we assume that $k(\lambda,\lambda') = 0$ when $\lambda > \lambda'$). But there are a continuum of values ν, namely $\nu \in [-1,+1]$ for which Eq.(5) has a solution in the distribution sense [2, 3]

$$\varphi_{\nu,(\zeta)}(\mu,\lambda) = \frac{\nu m(\nu,\zeta;\mu,\lambda)}{\nu-\mu} \tag{6}$$

$$+\left(\delta(\zeta-\lambda) - \nu\int_{-1}^{+1}\frac{\nu m(\nu,\zeta;\mu',\lambda)}{\nu-\mu'}d\mu'\right)\delta(\nu-\mu)$$

where $m(\nu,\zeta;\mu,\lambda)$ is the solution of the following integral equation

$$m(\nu,\zeta;\mu,\lambda)=k(\mu,\lambda;\nu,\zeta)$$
$$+\nu\int_a^b\int_{-1}^{+1}\frac{k(\mu,\lambda;\mu',\lambda')-k(\mu,\lambda;\nu,\lambda')}{\nu-\mu'}m(\nu,\zeta;\mu',\lambda')d\mu'd\lambda'$$
$$\nu,\mu\in[-1,+1]\quad \zeta,\lambda\in[a,b].$$

This equation admits unique continuous solution which can represented in the form

$$m(\nu,\zeta;\mu,\lambda)=\sum_{s=0}^{n}k_s(\lambda,\lambda')p_s(\mu)h_s(\nu,\zeta,\lambda)$$

where $h_s(\nu,\zeta,\lambda)$ is defined from the following recurrent relation

$$\nu h_s(\nu,\zeta,\lambda)-\frac{s+1}{2s+1}h_{s+1}(\nu,\zeta,\lambda)-\frac{s}{2s+1}h_{s-1}(\nu,\zeta,\lambda)$$
$$=\nu\int_a^b k_s(\lambda,\lambda')h_s(\nu,\zeta,\lambda')d\lambda'+k_s(\lambda,\zeta)p_s(\nu),$$

here $s=\overline{0,n},\quad h_0(\nu,\zeta,\lambda)=0.$

Along with Eq.(5) we will consider the following characteristic equation

$$(\nu-\mu)\varphi^*_\nu(\mu,\lambda)=\nu\int_a^b\int_{-1}^{+1}\sum_{s=0}^{n}(2s+1)k_s(\lambda',\lambda)p_s(\mu)p_s(\mu')\varphi^*_\nu(\mu',\lambda')d\mu'd\lambda'. \quad (7)$$

The singular eigenfunctions of Eq.(7) have the form

$$\varphi^*_{\nu,(\zeta)}(\mu,\lambda)=\frac{\nu m^*(\nu,\zeta;\mu,\lambda)}{\nu-\mu}$$
$$+\left(\delta(\zeta-\lambda)-\nu\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu',\lambda)}{\nu-\mu'}d\mu'\right)\delta(\nu-\mu)$$

where $m^*(\nu,\zeta;\mu,\lambda)$ is the solution of the following integral equation

$$m^*(\nu,\zeta;\mu,E)=k(\nu,\zeta;\mu,\lambda)+$$
$$\nu\int_a^b\int_{-1}^{+1}\frac{k(\mu',\lambda';\mu,\lambda)-k(\nu,\lambda';\mu,\lambda)}{\nu-\mu'}m^*(\nu,\zeta;\mu',\lambda')d\mu'd\lambda'$$
$$\mu,\nu\in[-1,+1],\quad \zeta,\lambda\in[a,b].$$

This equation also admits unique continuous solution which can de represented in the form

$$m^*(\nu,\zeta;\mu,\lambda)=\sum_{s=0}^{n}k_s(\lambda',\lambda)p_s(\mu)h^*_s(\nu,\zeta,\lambda)$$

where $h^*_s(\nu,\zeta,\lambda)$ is defined from the following recurrent relation

$$\nu h^*_s(\nu,\zeta,\lambda)-\frac{s+1}{2s+1}h^*_{s+1}(\nu,\zeta,\lambda)-\frac{s}{2s+1}h^*_{s-1}(\nu,\zeta,\lambda)$$
$$=\nu\int_a^b k_s(\lambda',\lambda)h^*_s(\nu,\zeta,\lambda')d\lambda'+k_s(\zeta,\lambda)p_s(\nu),$$

here $s=\overline{0,n},\quad h^*_0(\nu,\zeta,\lambda)=0.$

3 Some Properties of the Singular Eigenfunctions

The singular eigenfunctions of the characteristic equations have the following properties:

$$\int_{-1}^{+1} \varphi_{\nu,(\zeta)}(\mu,\lambda)d\mu = \delta(\zeta-\lambda) \tag{8}$$

and

$$\int_a^b \int_{-1}^{+1} \varphi_{\nu,(\zeta)}(\mu,\lambda)d\mu d\lambda = 1. \tag{9}$$

For the singular eigenfunctions $\varphi^*_{\nu,(\zeta)}(\mu,\lambda)$ we also have

$$\int_{-1}^{+1} \varphi^*_{\nu,(\zeta)}(\mu,\lambda)d\mu = \delta(\zeta-\lambda)$$

and

$$\int_a^b \int_{-1}^{+1} \varphi^*_{\nu,(\zeta)}(\mu,\lambda)d\mu d\lambda = 1.$$

The systems of the eigenfunctions represent the biorthogonal systems. The following equality

$$\int_a^b \int_{-1}^{+1} \mu\varphi^*_{\nu}(\mu,\lambda)\varphi_{\nu'}(\mu,\lambda)d\mu d\lambda = 0, \quad \nu \neq \nu'$$

holds.

Really, it is seen that the function $\varphi_{\nu,(\zeta)}(\mu,\lambda)$ satisfies the equation

$$\left(1-\frac{\mu}{\nu}\right)\varphi_{\nu}(\mu,\lambda) = \int_a^b \int_{-1}^{+1} k(\mu,\lambda;\mu',\lambda')\varphi_{\nu}(\mu',\lambda')d\mu' d\lambda'. \tag{10}$$

For the eigenfunctions $\varphi^*_{\nu,(\zeta)}(\mu,\lambda)$ we also have

$$\left(1-\frac{\mu}{\nu'}\right)\varphi^*_{\nu'}(\mu,\lambda) = \int_a^b \int_{-1}^{+1} k(\mu',\lambda';\mu,\lambda)\varphi^*_{\nu'}(\mu',\lambda')d\mu' d\lambda'. \tag{11}$$

If we multiply (8) by $\varphi^*_{\nu'}(\mu,\lambda)$, (9) by $\varphi_{\nu}(\mu,\lambda)$, subtract from the second equality the first and integrate we obtain

$$\left(\frac{1}{\nu}-\frac{1}{\nu'}\right)\int_a^b \int_{-1}^{+1} \mu\varphi^*_{\nu}(\mu,\lambda)\varphi_{\nu'}(\mu,\lambda)d\mu d\lambda = 0.$$

Also, it is truth the important equality for these singular eigenfunctions. Namely,

$$\int_a^b \int_{-1}^{+1} \mu\varphi^*_{\nu,(\zeta)}(\mu,\lambda)\varphi_{\nu',(\zeta')}(\mu,\lambda)d\mu d\lambda$$

$$= \nu\delta(\nu-\nu')\int_a^b \left(\delta(\zeta-\lambda) - \nu\int_{-1}^{+1} \frac{m^*(\nu,\zeta;\mu,\lambda)}{\nu-\mu}d\mu\right)$$

$$\times\left(\delta(\zeta'-\lambda)-\nu\int_{-1}^{+1}\frac{m(\nu',\zeta';\mu,\lambda)}{\nu'-\mu}d\mu\right)d\lambda.$$

Really, we have

$$\int_a^b\int_{-1}^{+1}\mu\varphi^*_{\nu,(\zeta)}(\mu,\lambda)\varphi_{\nu',(\zeta')}(\mu,\lambda)d\mu d\lambda$$

$$=\int_a^b\int_{-1}^{+1}\mu\Bigg(\frac{\nu m^*(\nu,\zeta;\mu,\lambda)}{\nu-\mu}$$

$$+\big(\delta(\zeta-\lambda)-\int_{-1}^{+1}\frac{\nu m^*(\nu,\zeta;\mu',\lambda)}{\nu-\mu'}d\mu'\big)\delta(\nu-\mu)\Bigg)$$

$$\times\Bigg(\frac{\nu' m(\nu',\zeta';\mu,\lambda)}{\nu'-\mu}$$

$$+\big(\delta(\zeta'-\lambda)-\int_{-1}^{+1}\frac{\nu' m(\nu',\zeta';\mu',\lambda)}{\nu'-\mu'}d\mu'\big)\delta(\nu'-\mu)\Bigg)d\mu d\lambda$$

$$=\int_a^b\int_{-1}^{+1}\mu\frac{\nu m^*(\nu,\zeta;\mu,\lambda)}{\nu-\mu}\frac{\nu' m(\nu',\zeta';\mu,\lambda)}{\nu'-\mu}d\mu d\lambda$$

$$+\frac{\nu m^*(\nu,\zeta;\nu',\lambda)}{\nu-\nu'}(\delta(\zeta-\lambda)-\int_{-1}^{+1}\frac{\nu m^*(\nu,\zeta;\mu',\lambda)}{\nu-\mu'}d\mu')d\lambda$$

$$+\int_a^b\frac{\nu' m(\nu',\zeta';\nu',\lambda)}{\nu-\nu'}(\delta(\zeta-\lambda)-\int_{-1}^{+1}\frac{\nu' m(\nu',\zeta';\mu',\lambda)}{\nu-\mu'}d\mu')d\lambda$$

$$+\nu\delta(\nu-\nu')\int_a^b(\delta(\zeta-\lambda)-\int_{-1}^{+1}\frac{\nu m^*(\nu,\zeta;\mu,\lambda)}{\nu-\mu}d\mu)$$

$$\times(\delta(\zeta'-\lambda)-\int_{-1}^{+1}\frac{\nu' m(\nu',\zeta';\mu,\lambda)}{\nu'-\mu}d\mu)d\lambda$$

By using the equality

$$\frac{\mu}{(\nu-\mu)(\nu'-\mu)}=\left(\frac{\nu}{\nu-\mu}-\frac{\nu'}{\nu'-\mu}\right)\frac{1}{\nu'-\nu}$$

we can write

$$\int_a^b\int_{-1}^{+1}\mu\frac{\nu m^*(\nu,\zeta;\mu,\lambda)}{\nu-\mu}\frac{\nu' m(\nu',\zeta';\mu,\lambda)}{\nu'-\mu}d\mu d\lambda$$

$$=\frac{\nu^2\nu'}{\nu'-\nu}\int_a^b\int_{-1}^{+1}\frac{\nu m^*(\nu,\zeta;\mu,\lambda)m(\nu'\zeta';\mu,E)}{\nu-\mu}d\mu d\lambda$$

$$-\frac{\nu\nu'^2}{\nu'-\nu}\int_a^b\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu,\lambda)m(\nu'\zeta';\mu,\lambda)}{\nu'-\mu}d\mu d\lambda$$

The singular eigenfunctions $\varphi^*_{\nu,(\zeta)}(\mu,\lambda)$ and $\varphi_{\nu',(\zeta')}(\mu,\lambda)$ satisfy the characteristic equations respectively and we have equalities

$$m(\nu',\zeta';\mu,\lambda)=\int_a^b\int_{-1}^{+1}k(\mu,\lambda;\mu',\lambda')\varphi_{\nu',(\zeta')}(\mu',\lambda')d\mu' d\lambda'$$

and

$$m^*(\nu,\zeta;\mu,\lambda)=\int_a^b\int_{-1}^{+1}k(\mu',E';\mu',E')\varphi^*_\nu(\mu',\lambda')d\mu'd\lambda'.$$

Multiply these equalities by $\varphi^*_{\nu,(\zeta)}(\mu,\lambda)$ and by $\varphi_{\nu',(\zeta')}(\mu,\lambda)$ respectively, subtract and integrate their with respect to μ and λ , we obtain

$$\int_a^b\int_{-1}^{+1}m(\nu',\zeta';\mu,\lambda)\varphi^*_{\nu,(\zeta)}(\mu,\lambda)d\mu d\lambda$$

$$-\int_a^b\int_{-1}^{+1}m^*(\nu,\zeta;\mu,\lambda)\varphi_{\nu',(\zeta')}(\mu,\lambda)d\mu d\lambda$$

$$=\int_a^b\int_{-1}^{+1}\varphi^*_{\nu,(\zeta)}(\mu,\lambda)\int_a^b\int_{-1}^{+1}k(\mu,\lambda;\mu',\lambda')\varphi_{\nu',(\zeta')}(\mu',\lambda')d\mu'd\lambda'd\mu d\lambda$$

$$-\int_a^b\int_{-1}^{+1}\varphi_{\nu',(\zeta')}(\mu,\lambda)\int_a^b\int_{-1}^{+1}k(\mu',\lambda';\mu,\lambda)\varphi^*_{\nu,(\zeta)}(\mu',\lambda')d\mu'd\lambda'd\mu d\lambda=0.$$

Consequently

$$\int_a^b\int_{-1}^{+1}m(\nu',\zeta';\mu,\lambda)\varphi^*_{\nu,(\zeta)}(\mu,\lambda)d\mu d\lambda$$

$$=\int_a^b\int_{-1}^{+1}m^*(\nu,\zeta;\mu,\lambda)\varphi_{\nu',(\zeta')}(\mu,\lambda)d\mu d\lambda.$$

Substitute in this equality expression for the eigenfunctions we have

$$\nu\int_a^b\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu,\lambda)m(\nu'\zeta';\mu,\lambda)}{\nu-\mu}d\mu d\lambda$$

$$+\int_a^b m(\nu',\zeta';\nu,\lambda)\left(\delta(\zeta-\lambda)-\nu\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu',\lambda)}{\nu-\mu'}d\mu'\right)d\lambda$$

$$=\nu\int_a^b\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu,\lambda)m(\nu'\zeta';\mu,\lambda)}{\nu'-\mu}d\mu d\lambda$$

$$+\int_a^b m^*(\nu,\zeta;\nu',\lambda)\left(\delta(\zeta'-\lambda)-\nu\int_{-1}^{+1}\frac{m(\nu',\zeta';\mu',\lambda)}{\nu'-\mu'}d\mu'\right)d\lambda.$$

From this equality we have

$$\nu\int_a^b\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu,\lambda)m(\nu'\zeta';\mu,\lambda)}{\nu-\mu}d\mu d\lambda$$

$$-\nu'\int_a^b\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu,\lambda)m(\nu'\zeta';\mu,\lambda)}{\nu'-\mu}d\mu d\lambda$$

$$=\int_a^b m^*(\nu,\zeta;\nu',\lambda)\left(\delta(\zeta'-\lambda)-\nu\int_{-1}^{+1}\frac{m(\nu',\zeta';\mu',\lambda)}{\nu'-\mu'}d\mu'\right)d\lambda$$

$$-\int_a^b m(\nu',\zeta';\nu,\lambda)\big(\delta(\zeta-\lambda)-\nu'\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu',\lambda)}{\nu-\mu'}d\mu'\big)d\lambda.$$

Using this equality we obtain

$$\nu\nu'\int_a^b\int_{-1}^{+1}\mu\frac{m^*(\nu,\zeta;\mu,\lambda)}{\nu-\mu}\frac{m(\nu',\zeta';\mu,\lambda)}{\nu'-\mu}d\mu d\lambda$$

$$=\frac{\nu\nu'}{\nu-\nu'}\Big(\int_a^b m^*(\nu,\zeta;\nu',\lambda)\big(\delta(\zeta'-\lambda)-\nu\int_{-1}^{+1}\frac{m(\nu',\zeta';\mu',\lambda)}{\nu'-\mu'}d\mu'\big)d\lambda$$
$$-\int_a^b m(\nu',\zeta';\nu,\lambda)\big(\delta(\zeta-\lambda)-\nu\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu',\lambda)}{\nu-\mu'}d\mu'\big)d\lambda\Big).$$

Therefore we have

$$\int_a^b\int_{-1}^{+1}\mu\varphi^*_{\nu,(\zeta)}(\mu,\lambda)\varphi_{\nu',(\zeta')}(\mu,\lambda)d\mu d\lambda$$

$$=\frac{\nu\nu'}{\nu-\nu'}\int_a^b m^*(\nu,\zeta;\nu',\lambda)\big(\delta(\zeta'-\lambda)-\nu'\int_{-1}^{+1}\frac{m(\nu',\zeta';\mu',\lambda)}{\nu'-\mu'}d\mu'\big)d\lambda$$
$$-\frac{\nu\nu'}{\nu-\nu'}\int_a^b m(\nu',\zeta';\nu',\lambda)\big(\delta(\zeta'-\lambda)-\nu\int_{-1}^{+1}\frac{m*(\nu,\zeta;\mu',\lambda)}{\nu'-\mu'}d\mu'\big)d\lambda$$
$$-\frac{\nu\nu'}{\nu-\nu'}\int_a^b m(\nu',\zeta';\nu',\lambda)\big(\delta(\zeta'-\lambda)-\nu\int_{-1}^{+1}\frac{m*(\nu,\zeta;\mu',\lambda)}{\nu'-\mu'}d\mu'\big)d\lambda$$
$$-\frac{\nu\nu'}{\nu-\nu'}\int_a^b m^*(\nu,\zeta;\nu',\lambda)\big(\delta(\zeta'-\lambda)-\nu'\int_{-1}^{+1}\frac{m(\nu',\zeta';\mu',\lambda)}{\nu'-\mu'}d\mu'\big)d\lambda$$
$$+\nu\delta(\nu-\nu')\int_a^b\left(\delta(\zeta-\lambda)-\nu\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu,\lambda)}{\nu-\mu}d\mu\right)$$
$$\times\left(\delta(\zeta'-\lambda)-\nu'\int_{-1}^{+1}\frac{m(\nu',\zeta';\mu,\lambda)}{\nu'-\mu}d\mu\right)d\lambda$$

and consequently we obtain

$$\int_a^b\int_{-1}^{+1}\mu\varphi^*_{\nu,(\zeta)}(\mu,\lambda)\varphi_{\nu',(\zeta')}(\mu,\lambda)d\mu d\lambda \tag{12}$$
$$=\nu\delta(\nu-\nu')\int_a^b\left(\delta(\zeta-\lambda)-\nu\int_{-1}^{+1}\frac{m^*(\nu,\zeta;\mu,\lambda)}{\nu-\mu}d\mu\right)$$
$$\times\left(\delta(\zeta'-\lambda)-\nu'\int_{-1}^{+1}\frac{m(\nu',\zeta';\mu,\lambda)}{\nu'-\mu}d\mu\right)d\lambda.$$

It is clear that the function

$$\tilde{\varphi}^*_{\nu,(\zeta)}(\mu,\lambda)=\varphi^*_{\nu,(\zeta)}(\mu,\lambda)+\int_a^b g(\nu,\zeta_0,\zeta')\varphi^*_{\nu,(\zeta')}(\mu,\lambda)d\zeta'$$

where $g(\nu,\zeta_0,\zeta)$ is an arbitrary continuous function, is also the singular eigenfunction of the characteristic equation (7). From (12) it follows that

$$\delta(\mu-\mu_0)\delta(\lambda-\lambda_0)=\mu\int_a^b\int_{-1}^{+1}\varphi_{\nu,(\zeta)}(\mu,\lambda)\tilde{\varphi}^*_{\nu,(\zeta)}(\mu_0,\lambda_0)d\nu d\zeta$$

$$\mu,\mu_0\in(-1,+1),\quad \lambda,\lambda_0\in[a,b]$$

where $g(\nu,\zeta_0,\zeta)$ is the unique solution of the integral equation

$$g(\nu,\zeta_0,\zeta)-\int_a^b S(\nu,\zeta',\zeta)g(\nu,\zeta_0,\zeta')d\zeta'=S(\nu,\zeta_0,\zeta)$$

and

$$S(\nu,\zeta_0,\zeta)=\int_{-1}^{+1}\frac{\nu m(\nu,\zeta;\mu,\zeta_0)}{\nu-\mu}d\mu+\int_{-1}^{+1}\frac{\nu m^*(\nu,\zeta_0;\mu,\zeta)}{\nu-\mu}d\mu$$
$$-\int_a^b\int_{-1}^{+1}\frac{\nu m(\nu,\zeta;\mu,\lambda)}{\nu-\mu}d\mu\int_{-1}^{+1}\frac{\nu m^*(\nu,\zeta_0;\mu,\lambda)}{\nu-\mu}d\mu d\lambda$$
$$-\pi^2\nu^2\int_a^b m(\nu,\zeta;\nu,\lambda)m^*(\nu,\zeta_0;\nu,\lambda)d\lambda,\quad \nu\in(-1,+1)\qquad \zeta,\zeta_0\in[a,b].$$

4 Green's Function

In order that satisfy the condition (3) we shall seek the solution on the form

$$G=\int_a^b\int_0^1 u(\nu,\zeta)\exp\left(-\frac{x-x_0}{\nu}\right)\varphi_{\nu,(\zeta)}(\mu,\lambda)d\nu d\zeta,\qquad x>x_0 \tag{13}$$

$$G=-\int_a^b\int_{-1}^0 u(\nu,\zeta)\exp\left(-\frac{x-x_0}{\nu}\right)\varphi_{\nu,(\zeta)}(\mu,\lambda)d\nu d\zeta,\qquad x<x_0 \tag{14}$$

where $\varphi_{\nu,(\zeta)}(\mu,\lambda)$, $\nu\in(-1,+1)$, $\zeta\in[a,b]$, is the singular eigenfunction of the characteristic equation (5), which is normalized as follows (9) and is represented in the following form (6) [2], $u(\nu,\zeta)$ is an unknown function. Our aim is to construct this function. When $x\to x_0$ the formulas (13),(14) turn into the following formulas

$$G^+(x_0,\mu_0,\lambda_0;x_0,\mu,\lambda)=\int_a^b\int_0^1 u(\nu,\zeta)\varphi_{\nu,(\zeta)}(\mu,\lambda)d\nu d\zeta \tag{15}$$

and

$$G^-(x_0,\mu_0,\lambda_0;x_0,\mu,\lambda)=-\int_a^b\int_{-1}^0 u(\nu,\zeta)\varphi_{\nu,(\zeta)}(\mu,\lambda)d\nu d\zeta. \tag{16}$$

Thus, the condition of jumping (3) reduced to the condition

$$\delta(\mu-\mu_0)\delta(\lambda-\lambda_0)=\mu\int_a^b\int_{-1}^{+1}u(\nu,\zeta)\varphi_{\nu,(\zeta)}(\mu,\lambda)d\nu d\zeta. \tag{17}$$

Consequently we can write

$$u(\nu,\zeta;\mu_0,\lambda_0)=\tilde{\varphi}^*_{\nu,(\zeta)}(\mu_0,\lambda_0)$$

Therefore, now we can write

$$G(x_0,\mu_0,\lambda_0;x,\mu,\lambda) \tag{18}$$
$$=\int_a^b\int_0^1 \exp\left(-\frac{x-x_0}{\nu}\right)\varphi_{\nu,(\zeta)}(\mu,\lambda)\tilde{\varphi}^*_{\nu,(\zeta)}(\mu_0,\lambda_0)d\nu d\zeta,\quad x>x_0$$

and

$$G(x_0,\mu_0,\lambda_0;x,\mu,\lambda) \tag{19}$$
$$=\int_a^b\int_{-1}^0 \exp\left(-\frac{x-x_0}{\nu}\right)\varphi_{\nu,(\zeta)}(\mu,\lambda)\tilde{\varphi}^*_{\nu,(\zeta)}(\mu_0,\lambda_0)d\nu d\zeta,\quad x<x_0$$

If we apply the normalization condition for $\varphi_{\nu,(\zeta)}(\mu,\lambda)$ and $\varphi^*_{\nu,(\zeta)}(\mu,\lambda)$ then for the neutrons faze and space density we obtain the results by averaging from (18),(19)

$$G(x_0,\mu_0,\lambda_0;x,\lambda)=\int_{-1}^{+1}G(x_0,\mu_0,\lambda_0;x,\mu,\lambda)d\mu$$

whence

$$G(x_0,\mu_0,\lambda_0;x,\lambda)=\int_0^1\tilde{\varphi}^*_{\nu,(\lambda)}(\mu_0,\lambda_0),\qquad x>x_0$$

and

$$G(x_0,\mu_0,\lambda_0;x,\lambda)=\int_{-1}^0\tilde{\varphi}^*_{\nu,(\lambda)}(\mu_0,\lambda_0),\qquad x<x_0.$$

For the density we have

$$\rho(x_0;x)=\int_a^b\int_{-1}^1\int_a^b\int_{-1}^1 G(x_0,\mu_0,\lambda_0;x,\mu,\lambda)d\mu_0 d\lambda_0 d\mu d\lambda.$$

In conclusion we can write

$$\rho(x_0;x)=\int_0^1\exp(-\mid x-x_0\mid/\nu)R(\nu)d\nu$$

where

$$R(\nu)=1+\int_a^b\int_a^b g(\nu,\zeta,\zeta')d\zeta' d\zeta.$$

Thus, the formulas for the research of the asymptotical properties of the phases density of the neutrons emitted from the single power source for the multivelocity case to some extent coincide with the corresponding known formulas for the one velocity case. Analysis of such functions are given in [3].

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