# APPROXIMATE SOLUTION OF PROBLEM STUDY FREE CONVECTION LAW CONDUCTIVE POWER-LAW FLUID

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*Abstract*

Approximate Solution of problem of steady free convection law conductive power-law fluid is given.

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Stationary free convection of the qualitative liquid and nontransparent heat transmitter and sufficiently electrical transmitter, which is caused by moving in flatness of the flat vertical endless disc is discussed.

The temperature of the disc is $T_W$, the speed of disc moving - $U_W$, liquid temperature far from disc is $T_\infty = const$, the speed of liquid pouring in the disc - $V_y = V_W = const$.

On any surface quality liquids and temperature boarding layer in outliet magnetic valley-equation has the form:

$$V_W \frac{\partial U}{\partial y} = \frac{nk}{\rho} \left| \frac{\partial U}{\partial y} \right|^{n-1} \frac{\partial^2 U}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_0^2}{\rho} U, \tag{1}$$

$$V_W \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2}. \tag{2}$$

Here $U$ is liquid speed to the direction of $x$ of axis, which coincides with the direction of disc moving. $g$ - is hastening of free falling, $\beta$ - the mark of heathy expansion; $\sigma$ - electrical transmission, $B_y = B_0$ - through the stagnant axis of $y$ by the magnetic vector induction, which is constant largeness.

Let's consider, that electrical transmission of liquid is changeable and represents the qualitative function of temperature

$$\sigma = \sigma_0 \left( \frac{T}{T_\infty} \right)^n, \qquad n \geq 0. \tag{3}$$

(1), (2) the explanation of equation system $U(y)$ and $T(y)$ must satisfy the following boarding conditions:

$$\begin{aligned} &U(y) = U_{_W} = const \ \text{ when } \ y = o, \\ &U(y) = 0 \qquad\qquad \text{ when } \ y = \delta, \\ &T(y) = T_{_W} = const \ \text{ when } \ y = o, \\ &T(y) = T_\infty \qquad\qquad \text{ when } \ y = \delta_r. \end{aligned} \tag{4}$$

The explanation of system, which are defined by the method of sequence approaching, has the following form:

$$\begin{aligned} &U(y) \approx U_0(y) + U_1(y) = U_{_W}\left(1 - \frac{y}{\delta}\right) \\ &+ \frac{\rho U_{_W}^{1-n}}{nk}(-\delta)^{n-1}\left[\left[V_{_W}U_{_W}(-\delta)^{-1} + g\beta(T_\infty - T_{_W})\right]\left(\frac{y^2}{2} - \frac{\delta y}{2}\right)\right. \\ &- \frac{g\beta(T_\infty - T_{_W})(y^3 - \delta^2 y)}{6\delta_r} + \frac{M_1 T_0^{n+2}(y)}{(n+1)(n+2)} - \frac{M_1 y T_0^{n+2}(\delta)}{\delta(n+1)(n+2)} \\ &- \frac{M_1\delta_T T_0^{n+3}(y)}{\delta(T_\infty - T_{_W})(n+2)(n+3)} + \frac{M_1\delta_T y T_0^{n+3}(\delta)}{\delta^2(T_\infty - T_{_W})(n+2)(n+3)} \\ &+ \frac{M_1 T_{_W}\delta_T T_0^{n+2}(y)}{\delta(T_\infty - T_{_W})(n+1)(n+2)} - \frac{M_1 T_{_W}\delta_T^2 T_0^{n+2}(\delta)}{\delta^2(T_\infty - T_{_W})(n+1)(n+2)} \\ &\left.- \frac{M_1 T_{_W}^{n+2}}{(n+1)(n+2)}\left(1 + \frac{2\delta_T T_{_W}(y)}{\delta(T_\infty - T_{_W})(n+3)}\right)\left(1 - \frac{y}{\delta}\right)\right], \end{aligned} \tag{5}$$

$$T(y) = \frac{T_\infty - T_{_W}}{\delta_T}y + T_{_W} + \frac{V_{_W}(T_\infty - T_{_W})}{2a}\left(\frac{y^2}{\delta_T} - y\right), \tag{6}$$

where

$$\begin{cases} T_0(y) = \dfrac{T_\infty - T_{_W}}{\delta_T}y + T_{_W}, \\ M_1 = \dfrac{MU_{_w}\delta_T^2}{T_\infty^n(T_\infty - T_{_W})^2}, \ \ M = \dfrac{\sigma_0 B_0^2}{\rho}, \ \ n \neq \dfrac{1}{2m}, \ \ m \in N \end{cases} \tag{7}$$

The thickness of boarding layer $\delta$ and warmth thickness $\delta_T$ are defined from the conditions:

$$\left.\frac{\partial U}{\partial y}\right|_{y=\delta} = 0, \quad \left.\frac{\partial T}{\partial y}\right|_{y=\delta_T} = 0. \tag{8}$$

For $\delta_T$- we'll get the definition -

$$\delta_T = -\frac{2a}{V_W}, \tag{9}$$

so for $\delta$ - we'll get -

$$1 - \frac{\rho U_W^{-n}}{nk}(-\delta)^{n+1}\left[\frac{1}{2}\left[\frac{V_W U_W}{-\delta^2} + g\beta(T_\infty - T_W)\right] - \frac{g\beta(T_\infty - T_W)}{3}\cdot\frac{\delta}{\delta_T}\right.$$
$$+\frac{MU_w}{T_\infty^n(T_\infty - T_W)(n+1)}\frac{\delta_T}{\delta}\left(1 + \frac{\delta_T T_W}{\delta(T_\infty - T_W)}\right)T_0^{n+1}(\delta)$$

$$-\frac{MU_W}{T_\infty^n(T_\infty - T_W)^2(n+1)(n+2)}\frac{\delta_T^2}{\delta^2}\left(1 + \frac{T_W}{T_\infty - T_W}\right)T_0^{n+2}(\delta)$$
$$-\frac{MU_W}{(T_\infty - T_W)^2(n+2)}\frac{\delta_T^2}{\delta^2}T_0^{n+2}(\delta) \tag{10}$$
$$+\frac{MU_W}{T_\infty^n(T_\infty - T_W)^2(n+2)(n+3)}\cdot$$

$$\left.\tfrac{\delta_T^3}{\delta^3}T_0^{n+3}(\delta)\right]$$

$$-\frac{\rho U_n^{1-n}(-\delta)^{n-1}M\delta_T^2 T_W^{n+2}}{nkT_\infty^n(T_\infty - T_W)^2(n+1)(n+2)}\left[1 + \frac{2\delta_T T_W}{\delta(T_\infty - T_W)(n+3)}\right] = 0.$$

Let's discuss two cases, when the explanation of (10) is given by the explicit form:

1) $\dfrac{\delta}{\delta_T} = \alpha = const$; 2) $\delta = \dfrac{T_W\delta_T}{T_\infty - T_W}$.

In the first case, it's received that

$$\delta = \pm\left[\frac{(\rho V_W + 2k)U_W}{2\rho\alpha_1}\right]^{\frac{1}{2}}, \quad \text{when} \quad n = 1, \tag{11}$$

and

$$\delta = \pm\left[\frac{\rho V_W U_W \pm (\rho^2 V_W^2 U_W^4 + 48\rho\alpha_1 k U_W^3)^{\frac{1}{2}}}{2\rho\alpha_1}\right]^{\frac{1}{2}}, \quad \text{when} \quad n = 3, \tag{12}$$

where $\alpha_1$ is a known constant

$$\alpha_1 = g\beta(T_\infty - T_W)\left(\frac{1}{2} - \frac{\alpha}{3}\right) + \frac{MU_W\left((T_\infty - T_W)\alpha + T_W\right)^{n+1}}{T_\infty^n (T_\infty - T_W)(n+1)\alpha}$$
$$\times\left(1 + \frac{T_W}{\alpha(T_\infty - T_W)}\right)\left[1 - \frac{(T_\infty - T_W)\alpha + T_W}{(T_\infty - T_W)(n+2)\alpha}\right]$$
$$-\frac{MU_W\left((T_\infty - T_W)\alpha + T_W\right)^{n+2}}{T_\infty^n (T_\infty - T_W)^2(n+2)\alpha^2}\left[1 - \frac{(T_\infty - T_W)\alpha + T_W}{(T_\infty - T_W)(n+3)\alpha}\right]$$
$$-\frac{U_w M_W^{n+2}}{T_\infty^n (T_\infty - T_W)^2(n+1)(n+2)\alpha^2}\left(1 + \frac{2T_W}{\alpha(T_\infty - T_W)(n+3)}\right).$$

In another case the explanation of equation (10) is given by the following form:

$$\delta = \pm\left[\frac{(\rho(V_W - c_4) + 2k)}{2\rho g\beta U_W^{-3}\left(T_\infty - \frac{T_W}{3}\right)}\right]^{\frac{1}{2}}, \quad \text{when} \quad n = 1, \tag{13}$$

and

$$\delta = \pm\left[\frac{\rho U_W^{-2}(V_W - c_4) \pm \left[\rho^2 V_W^{-4}(V_W - c_4) + 24k\rho U_W^{-3} g\beta\left(T_\infty - \frac{T_W}{3}\right)\right]^{\frac{1}{2}}}{2\rho g\beta U_W^{-3}\left(T_\infty - \frac{T_W}{3}\right)}\right]^{\frac{1}{2}},$$

$$\text{when } n = 3, \tag{14}$$

where $c_4$ is a known constant

$$c_4 = \frac{2M\delta_T^2 T_W^{n+2}}{T_\infty^n (T_\infty - T_W)^2(n+1)(n+2)}\left(1 + \frac{2}{n+3}\right).$$

For surface friction $r = k\left|\frac{\partial U}{\partial y}\right|_{y=0}^n$ and friction of $c_r = \frac{r}{\rho U_W}$ - we'll get the definition

$$\tau = kG^n, \tag{15}$$

$$\tau = \frac{kG^n}{\rho U_W}. \tag{16}$$

where $G$ is constant largeness and it's calculated by the formula:

$$
\begin{aligned}
G = &-\frac{U_W}{\delta} + \frac{\rho U_W^{1-n}}{nk}(-\delta)^{n-1}\left[\left[V_W U_W \delta^{-1} + g\beta(T_\infty - T_W)\right]\frac{\delta}{2}\right.\\
&+\frac{g\beta}{6}\cdot\frac{T_\infty - T_W}{\delta_t}\delta^2 + \frac{MU_W\delta_T(\delta T_\infty - T_W(\delta-\delta_T))}{\delta T_\infty^n (T_\infty - T_W)(n+1)}T_W^{n+1}\\
&-\frac{MU_W\delta_T^2(\delta + T_W\delta_r)}{\delta^2 T_\infty^n (T_\infty - T_W)^2(n+1)(n+2)}T_0^{n+2}(\delta)\\
&+\frac{MU_W\delta_T^3 T_0^{n+3}(\delta)}{\delta^2 T_\infty^n (T_\infty - T_W)^3(n+2)(n+3)}\\
&\left.+\frac{M\delta_T^2 T_W^{n+2} U_W}{\delta T_\infty^n (T_\infty - T_W)^2(n+1)(n+2)}\left(\frac{2\delta_T T_W}{\delta(T_\infty - T_W)(n+3)} - n\right)\right].
\end{aligned}
\tag{17}
$$

in which $T_0(\delta)$ and $M_1$ is calculated from (8).

*References*


1. J. V. Sharikadze, Application of the Method Slezkina-Tag for the approximate Solution of the boundary Layer of a conducting Fluid with a variable Coefficient of Conductivity. *Fifth Congress of Mathematicians of Georgia Abstracts of contributed Talks*, Batumi/Kutaisi, October 9-12, 2009.

2. Kh. Mshvenierdze, N. Kobadze, Hydrodynamic Flow in a Periodic Boundary Layer in the Case of variable Infiltration and variable Conductance. *Fifth Congress of Mathematicians of Georgia Abstracts of contributed Talks*, Batumi/Kutaisi, October 9-12, 2009.

3. N. Kobadze, Kh. Mshvenieradze, Pulsate Flow and Heat transfer of a Conducting Fluid in a porous Channel with a given external Magnetic Field. *Fifth Congress of Mathematicians of Georgia Abstracts of contributed Talks*, Batumi/Kutaisi, October 9-12, 2009.