ON THE QUADRATIC FORM OF TYPE (-2, q, 1) WITH DISCRIMINANT q^2

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Abstract

The quadratic form of type (-2, q, 1) are derived. Explicit formulas are obtained for $q \equiv -1(mod6)$. These quadratic forms are reduced. Then it is shown how formulae can be obtained for the number of representations of positive integers by means the constructed quadratic forms.

Key words and phrases: Positive quadratic forms, quadratic residue, reduction of quadratic forms, spherical polynomial, generalized theta-series.

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1 Introduction

Let

$$Q(x) = Q(x_1, \cdots, x_f) = \sum_{0 \le r \le s \le f} b_{rs} x_r x_s \tag{1.1}$$

be an integer positive definite quadratic form in an even number f of variables. That is, $b_{rs} \in \mathbb{Z}$ and $Q(x) > 0$ if $x \neq 0$. To $Q(x)$ we associate the even integral symmetric $f \times f$ matrix A defined by $a_{rr} = 2b_{rr}$ and $a_{rs} = a_{sr} = b_{rs}$, where $r \le s$. If $X = [x_1, \cdots, x_f]'$ denotes a column vector, where $'$ denotes the transpose, then we have $Q(x) = \frac{1}{2}X'AX$. Let A_{ij} denote the cofactor to the element a_{ij} in $D = \det A$ and a^*_{ij} the corresponding element of A^{-1}. $\Delta = (-1)^{\frac{f}{2}} D$ denote the discriminant of the quadratic form $Q(x)$;

$$\delta = \gcd\,(\frac{1}{2}A_{rr}, A_{rs}) \qquad (r, s = 1, 2, \cdots, f),$$

$N = \frac{D}{\delta}$ is the step of quadratic form $Q(x)$; $\chi(d)$ is a character of quadratic form $Q(x)$, e.i. if Δ is square, then $\chi(d) = 1$, and if Δ is not square, then

$$\chi(d) = \begin{cases} (\frac{d}{|\Delta|}) & \text{if} \quad d > 0, \\ (-1)^{\frac{f}{2}} \chi(-d) & \text{if} \quad d < 0, \end{cases}$$

where $(\frac{d}{|\Delta|})$ is the generalized Jacobi symbol. A positive quadratic form of weight $\frac{f}{2}$, step N and character χ is called a quadratic form of type $(-\frac{f}{2}, N, \chi)$.

Below we shall use the notions, notation and some results from [1]. In the follows q is odd prime, $z = \exp(2\pi i\tau)$, $\operatorname{Im}\tau > 0$.

A homogeneous polynomial $P_\nu(x) = P_\nu(x_1, \cdots, x_f)$ of degree ν with complex coefficients, satisfying the condition

$$\sum_{1\le i,j\le f} a^*_{ij}\left(\frac{\partial^2 P}{\partial x_i\,\partial x_j}\right) = 0,$$

is called a spherical polynomial of order ν with respect to $Q(x)$ (see [2]).

It is known, that ([1], pp. 874, 817) if $Q(x)$ is a quadratic form of type $(-k, q, 1)$, $2|k$, $k > 2$, then the discriminant

$$\Delta = q^{2l} \qquad 1 \le l \le k-1 \tag{1.2}$$

and

$$E(\tau, Q(x)) = 1 + \sum_{n=1}^{\infty}(\alpha\,\sigma_{k-1}(n)z^n + \beta\sigma_{k-1}(n)z^{qn}) \tag{1.3}$$

is the corresponding Eisenstein series, where $\sigma_{k-1}(n) = \sum\limits_{d|n} d^{k-1}$ and

$$\alpha = \frac{i^k}{\rho_k}\,\frac{q^{k-l} - i^k}{q^k - 1}, \qquad \beta = \frac{1}{\rho_k}\,\frac{q^k - i^k q^{k-l}}{q^k - 1},$$

$$\rho_k = (-1)^{\frac{k}{2}}\frac{(k-1)!}{(2\pi)^k}\,\zeta(k), \qquad (\zeta(k) \text{ is theRiemann } \zeta\text{−function})$$

In particular,

$$\rho_4 = \frac{1}{240}.$$

For each positive quadratic form $Q(x)$

$$\vartheta(\tau, Q(x)) = 1 + \sum_{n=1}^{\infty} r(n, Q(x))\, z^n \tag{1.4}$$

is the corresponding theta-series, where $r(n, Q(x))$ denote the number of representation of the positive integer n by the quadratic form $Q(x)$.

Let quadratic form $Q(x)$ has a form (1.1) and

$$4b_{11}Q(x) = (2b_{11}x_1 + b_{12}x_2 + \cdots, b_{1F}x_f)^2 + G(x_2, \cdots, x_f).$$

Lemma 1. ([3], p.10) *The quadratic form $Q(x)$ is reduced by Hermite, if*

$$\min Q(x) = |b_{11}| > 0, \ |b_{1j}| \leq |b_{11}| \quad (j = 2, 3, \cdots, f)$$

and $G(x_2, \cdots, x_f)$ is reduced.

Lemma 2. ([1], p. 853) *Among homogenous quadratic polynomials in f variables*

$$\varphi_{ij} = x_i x_j - \frac{A_{ij}}{fD} 2Q(X) \quad (i, j = 1, ...f)$$

exactly $\frac{f(f+1)}{2} - 1$ ones are linearly independent and form the basis of the space of spherical polynomials of second order with respect to $Q(x)$.

Lemma 3. ([1], p. 808, 855) *Let $Q(x)$ be a positive reduced quadratic form of type $(-\frac{f}{2}, N, \chi), 2|k$, and $P_\nu(x)$ is a spherical polynomial of order ν with respect to $Q(x)$, then for $\nu > 0$ the generalized r-fold theta-series*

$$\vartheta(\tau, P_\nu(x), Q(x)) = \sum_{x \in \mathbb{Z}^{\upsilon}} P_\nu(x) z^{Q(x)} = \sum_{n=1}^{\infty} (\sum_{Q(x)=n} P_\nu(x)) z^n$$

is a cusp form of type $(-(\frac{f}{2} + \nu), N, \chi)$.

Lemma 4. ([1], p. 846) *Let quadratic forms $Q_1(x)$ and $Q_2(x)$ have the same step N and characters $\chi_1(d)$ and $\chi_2(d)$, respectively, then the quadratic form $Q_1(x) + Q_2(x)$ has the step N and character $\chi_1(d)\chi_2(d)$.*

Lemma 5. ([1], pp. 874, 875) *Let $Q(x)$ be a positive reduced quadratic form of type $(-k, q, 1), 2|k, k > 2$ then difference $\vartheta(\tau, Q(x)) - E(\tau, Q(x))$ is a cusp form of type $(-k, q, 1)$.*

It is known the some reduced quadratic forms of type $(-2, q, 1)$ with discriminant q^2, when $q = 3, 5, 7$ ([4]), $q = 11, 17$ ([1], pp. 901-902).

M. Eichler ([5] pp. 234-235) proof, that the cusp form of type $(-k, q, 1)$ is represented in the linear combination of generalized quaternary theta-series.

In this paper is also constructed the reduced quadratic forms of type $(-2, q, 1)$ with discriminant q^2 for every prime $q > 3$. By using these quadratic forms, the basis of spaces of cusp forms of type $(-4, q, 1)$ when $q = 13$ is constructed. Then formulae are derived for the number of representations of positive integers by the quadratic forms of corresponding types.

2 The construction of the quadratic form of type (-2, q, 1)

The matrix A and its determinant $D = detA$ of quadratic forms of type $(-2, q, 1)$ with discriminant q^2 must satisfy the following conditions:
1. The matrix A should be a fourth order symmetric matrix;
2. Its elements of main diagonal should be positive even numbers and other elements -
integer numbers;
3. All its principal minor should be positive;
4. $D = q^2$ and $\delta = \gcd\left(\frac{1}{2}A_{rr}, A_{rs}\right)_{r,s=1,2,3,4} = q$.

Let find D with form

$$\begin{vmatrix} 2 & 1 & b_{13} & 0 \\ 1 & 2b_{22} & 0 & 0 \\ b_{13} & 0 & 2b_{33} & q \\ 0 & 0 & q & 2q \end{vmatrix},$$

where b_{13}, b_{22} and b_{33} are integer numbers.

The determinant D satisfies the conditions 1-4 if

$$2q \begin{vmatrix} 2 & 1 & b_{13} \\ 1 & 2b_{22} & 0 \\ b_{13} & 0 & 2b_{33} \end{vmatrix} - q^2 \begin{vmatrix} 2 & 1 \\ 1 & 2b_{22} \end{vmatrix} = q^2$$

and

$$\begin{vmatrix} 2 & 1 \\ 1 & 2b_{22} \end{vmatrix} > 0.$$

It is sufficient to show that there exists an integer numbers b_{13}, b_{22} and b_{33} that

$$\begin{vmatrix} 2 & 1 & b_{13} \\ 1 & 2b_{22} & 0 \\ b_{13} & 0 & 2b_{33} \end{vmatrix} = 2b_{22}q, \ \ 4b_{22} > 1,$$

i.e.

$$b_{33}(4b_{22} - 1) - b_{11}^2 b_{22} = b_{22}q, \ \ b_{22} > 0. \tag{2.1}$$

From (2.1) it follows that $b_{22}|b_{33}$, i.e. $b_{33} = b_{22}t$ and condition (2.1) takes the form

$$t(4b_{22} - 1) - b_{13}^2 = q,$$

i.e.

$$b_{13}^2 \equiv -q(mod 4b_{22} - 1). \tag{2.2}$$

Thus the determinant D satisfies conditions 1-4 if we find the integer number $b_{22} > 0$, such that $4b_{22}-1$ will be prime and $(\frac{-q}{4b_{22}-1}) = 1$ (where $(\frac{-q}{4b_{22}-1})$ is the Legandre symbol and $(\frac{-q}{4b_{22}-1}) = -(\frac{q}{4b_{22}-1}) = -(-1)^{\frac{q-1}{2}}(\frac{4b_{22}-1}{q})$). If we solve the congruence (2.2), we get b_{13} and from (2.1) we obtain $b_{33} = \frac{b_{22}q+b_{13}^2 b_{22}}{4b_{22}-1}$.

Hence, our problem is following: Find the integer number $b_{22} > 0$, such that $4b_{22} - 1$ will be prime number and quadratic residue of q, if $q \equiv 3(mod 4)$, or quadratic nonresidue of q, if $q \equiv 1(mod 4)$.

We now prove the following

Lemma. *if $q \neq 3$ is an odd prime, then there are exactly $\frac{q-1}{2}$ natural numbers b_{22}, such that $4b_{22} - 1$ will be prime and*

$$(\frac{4b_{22}-1}{q}) = 1, \quad \text{if} \quad q \equiv 3(mod 4),$$

or

$$(\frac{4b_{22}-1}{q}) = -1, \quad \text{if} \quad q \equiv 1(mod 4).$$

.

Proof. a) Let $q \equiv 3(mod 4)$, consider the system of congruence

$$\begin{cases} x \equiv -1 \pmod 4 \\ x \equiv c \pmod q \end{cases} \tag{2.3}$$

where c is some quadratic residue of q. From the Chinese remainder theorem, there are exactly one solution x_0 modulo $4q$, $x_0 = -qy_1 + 4cy_2$, where y_1 is the solution of congruence $qy \equiv 1 \pmod 4$ and y_2 is the solution of congruence $4y \equiv 1 \pmod q$.

Consider now the arithmetic progression $x_0 + 4qt$. In this progression $(x_0, 4q) = 1$ and from the Dirichlet's Theorem on primes in arithmetic progressions there are infinitely many primes, where satisfies the system of congruence (2.3).

It is known that c can get $\frac{q-1}{2}$ different values, accordingly there exists $\frac{q-1}{2}$ incongruent integers modulo q, where satisfies the system (2.3).

b) Similarly consider the case $q \equiv 1(mod 4)$.In the system of congruence (2.3) c is nonquadratic residue of q. We get there exists $\frac{q-1}{2}$ incongruent integers modulo q, where satisfies the system (2.3). ■

Below the values of q and b_{22} for prime $q \leq 29$ are obtained: For $q = 5$, $b_{22} = 1, 2$;
For $q = 7$, $b_{22} = 3, 6, 11$;
For $q = 11$, $b_{22} = 1, 6, 8, 15, 18$;
For $q = 13$, $b_{22} = 2, 3, 5, 8, 12, 17$;
For $q = 17$, $b_{22} = 1, 2, 6, 8, 12, 27, 41, 50$;
For $q = 19$, $b_{22} = 2, 3, 6, 11, 12, 33, 35, 48, 66$;
For $q = 23$, $b_{22} = 1, 8, 12, 15, 18, 32, 33, 42, 45, 54, 60$;
For $q = 29$, $b_{22} = 1, 3, 5, 8, 11, 12, 20, 33, 48, 54, 68, 83, 123, 188$.

Construct now some quadratic forms of type $(-2, q, 1)$.

If $q = 13$ and $b_{22} = 2$, from (2.2) and (2.1) we have $b_{13} = \pm 1$, $b_{33} = 4$. Thus the quadratic form

$$Q_1(x) = x_1^2 + 2x_2^2 + 4x_3^2 + 13x_4^2 + x_1x_2 + x_1x_3 + 13x_3x_4$$

is the quadratic form of type $(-2, 13, 1)$ with discriminant 13^2.

If $q = 119$ and $b_{22} = 2$, from (2.2) and (2.1) we have $b_{13} = \pm 3$, $b_{33} = 8$. Thus the quadratic form

$$Q_2(x) = x_1^2 + 2x_2^2 + 8x_3^2 + 19x_4^2 + x_1x_2 + 3x_1x_3 + 19x_3x_4$$

is the quadratic form of type $(-2, 19, 1)$ with discriminant 19^2.

Consider now the particular case $q \equiv -1(mod6)$.

It is easily to verify that the determinant

$$\begin{vmatrix} 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 1 & \frac{2}{3}(q+) & q \\ 0 & 0 & q & 2q \end{vmatrix} = q^2$$

satisfies the condition 1-4, i.e.

$$Q_3(x) = x_1^2 + x_2^2 + \frac{q+1}{3}x_3^2 + qx_4^2 + x_1x_2 + x_1x_3 + x_2x_3 + qx_3x_4$$

is the quadratic form of type $(-2, q, 1)$ with discriminant q^2 (where $q \equiv -1(mod6)$).

3 The reduction of the constructed quadratic forms

Find now an equivalent reduced quadratic form of

$$Q_3(x) = x_1^2 + x_2^2 + \frac{q+1}{3}x_3^2 + qx_4^2 + x_1x_2 + x_1x_3 + x_2x_3 + qx_3x_4.$$

It is clear that

$$4Q_3(x) = 4x_1^2 + 4x_2^2 + \frac{4}{3}(q+1)x_3^2 + 4qx_4^2 + 4x_1x_2 + 4x_1x_3 + 4x_2x_3 + 4qx_3x_4 =$$

$$= (2x_1 + x_2 + x_3)^2 + G(x_2,\, x_3,\, x_4),$$

where

$$G(x_2,\, x_3,\, x_4) = 3x_2^2 + \frac{4q+1}{3}x_3^2 + 4qx_4^2 + 2x_2x_3 + 4qx_3x_4.$$

Similarly

$$12G(x_2,\, x_3,\, x_4) = (6x_2 + 2x_3)^2 + g(x_3, x_4),$$

where

$$g(x_3, x_4) = 16q(x_3^2 + 3x_3x_4 + 3x_4^2).$$

By using well-known algorithm ([6] p. 141-144), the linear transformation with matrix

$$\begin{Vmatrix} 1 & -1 \\ 0 & 1 \end{Vmatrix}$$

takes the quadratic form $g(x_3, x_4)$ into the equivalent reduced quadratic form. Now use the linear transformation with matrix

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

for quadratic form $Q_3(x)$, we get

$$Q(x) = x_1^2 + x_2^2 + \frac{q+1}{3}x_3^2 + \frac{q+1}{3}x_4^2 + x_1x_2 + x_1x_3 - x_1x_4 + x_2x_3 - x_2x_4 + \frac{q-2}{3}x_3x_4.$$

It is easy to verify that $Q(x)$ is a reduced quadratic form of type $(-2, q, 1)$ with discriminant q^2 for every prime number $q \equiv -1(mod 6)$).

Similarly by using the linear transformation with matrix

$$\begin{Vmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 0 \end{Vmatrix}$$

for quadratic form $Q_1(x)$, and the linear transformation with matrix

$$\begin{Vmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 0 \end{Vmatrix}$$

for quadratic form $Q_2(x)$, we obtain that

$$Q_4(x) = x_1^2 + 2x_2^2 + 2x_3^2 + 4x_4^2 + x_1x_2 + x_1x_4 - x_2x_3 + 2x_3x_4$$

is a reduced quadratic form of type $(-2, 13, 1)$ with discriminant 13^2 and

$$Q_5(x) = x_1^2 + 2x_2^2 + 3x_3^2 + 6x_4^2 + x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 - x_2x_4 + 3x_3x_4$$

is a reduced quadratic form of type $(-2, 19, 1)$ with discriminant 19^2.

4 The number of representations of the positive integer n by the quadratic form of type $(-4, q, 1)$

For quadratic form of type $(-2, 13, 1)$

$$Q_4(x) = x_1^2 + 2x_2^2 + 2x_3^2 + 4x_4^2 + x_1x_2 + x_1x_4 - x_2x_3 + 2x_3x_4$$

we have that $\Delta = D = 13^2, A_{11} = 104,\ A_{12} = -26,\ A_{13} = 0$.

According to the lemma 2 the spherical polynomials has the form

$$\begin{aligned}
&\varphi_{11} = x_1^2 - \tfrac{A_{11}}{4D}2Q_4 = x_1^2 - \tfrac{4}{13}Q_4,\\
&\varphi_{12} = x_1x_2 - \tfrac{A_{12}}{4D}2Q_4 = x_1x_2 + \tfrac{1}{13}Q_4,\\
&\varphi_{13} = x_1x_3.
\end{aligned}$$

Consider the equation

$$x_1^2 + 2x_2^2 + 2x_3^2 + 4x_4^2 + x_1x_2 + x_1x_4 - x_2x_3 + 2x_3x_4 = n. \tag{4.1}$$

For $n = 1$, equation has 2 solutions $x_1 = \pm 1$.

For $n = 2$, equation has 6 solutions $x_2 = \pm 1;\ x_3 = \pm 1;\ x_1 = \pm 1,\ x_2 = \mp 1$.

For $n = 3$, equation has 8 solutions $x_1 = \pm 1,\ x_3 = \pm 1;\ x_1 = \pm 1,\ x_3 = \mp 1;\ x_2 = \pm 1,\ x_3 = \pm 1;\ x_1 = \pm 1,\ x_2 = \mp 1,\ x_3 = \mp 1$. Here all other $x_i = 0$.

Using these solutions and performing easy calculations by lemma 3 we obtain

$$\vartheta(\tau, \varphi_{11}, Q_4) = \sum_{n=1}^{\infty}\Big(\sum_{Q_4=n} x_1^2 - \frac{4}{13}n\Big)z^n = \frac{18}{13}z - \frac{22}{13}z^2 - \frac{18}{13}z^3 + .\ \ .\ \ .\ ,$$

$$\vartheta(\tau, \varphi_{12}, Q_4) = \sum_{n=1}^{\infty}\Big(\sum_{Q_4=n} x_1x_2 + \frac{1}{13}n\Big)z^n = \frac{2}{13}z - \frac{14}{13}z^2 - \frac{2}{13}z^3 + .\ \ .\ \ .\ ,$$

$$\vartheta(\tau, \varphi_{13}, Q_4) = \sum_{n=1}^{\infty} \Big(\sum_{Q_4=n} x_1 x_3 \Big) z^n = -2z^3 + . \quad . \quad . \quad .$$

These generalized quaternary theta-series are the cusp form of type (-4, 13, 1), they are linearly independent, since the determinant constructed from the coefficients of these theta-series is not equal to zero. It is known ([1] p. 899) that the maximal number of linearly independent cusp form of type (-4, 13, 1) is 3. Thus we have proved

Theorem 1. The generalized quaternary theta-series

$$\begin{aligned} \vartheta(\tau, \varphi_{11}, Q_4) &= \textstyle\sum_{n=1}^{\infty} \big(\sum_{Q_4=n} x_1^2 - \frac{4}{13} n \big) z^n, \\ \vartheta(\tau, \varphi_{12}, Q_4) &= \textstyle\sum_{n=1}^{\infty} \big(\sum_{Q_4=n} x_1 x_2 + \frac{1}{13} n \big) z^n, \\ \vartheta(\tau, \varphi_{13}, Q_4) &= \textstyle\sum_{n=1}^{\infty} \big(\sum_{Q_4=n} x_1 x_3 \big) z^n \end{aligned} \tag{4.2}$$

form a basis of the space of cusp form of type (-4, 13, 1).

Consider the quadratic form

$$F = Q_4(x_1, x_2, x_3, x_4) + Q_4(x_5, x_6, x_7, x_8).$$

By lemma 4 and (1.2) we have $\Delta = D = 13^4, l = 2, N = q = 13, \chi(d) = 1$, i.e. the quadratic form F is a form of type $(-4, q, 1)$.

From (1.4) using the number of solutions of equation (4.1) we obtain that

$$\vartheta(\tau, Q_4) = 1 + \sum_{n=1}^{\infty} r(n, Q_4)\, z^n = 1 + 2z + 6z^2 + 8z^3 + . \quad . \quad .$$

hence

$$\vartheta(\tau, F) = \vartheta^2(\tau, Q_4) = 1 + 4z + 16z^2 + 40z^3 + . \quad . \quad . \tag{4.3}$$

if $q = 13$ and $l = 2$ from (1.3) we have

$$E(\tau, F) = 1 + \frac{24}{17} \sum_{n=1}^{\infty} \big(\sigma_3(n) z^n + 169 \sigma_3(n) z^{13n} \big)$$

$$= 1 + \frac{24}{17} z + \frac{24 \cdot 9}{17} z^2 + \frac{24 \cdot 28}{17} z^3 + \quad . \quad . \tag{4.4}$$

According to the Lemma 5, $\vartheta(\tau, F) - E(\tau, F)$ is a cusp form of type $(-4, 13, 1)$. Thus, from Theorem 1 there are constants c_1, c_2 and c_3 such that

$$\vartheta(\tau, F) - E(\tau, F) = c_1 \vartheta(\tau, \varphi_{11}, Q_4) + c_2 \vartheta(\tau, \varphi_{12}, Q_4) + c_3 \vartheta(\tau, \varphi_{13}, Q_4).$$

Equating the coefficients of $z,\ z^2$ and z^3 on the both sides of this identity, using (4.3), (4.4), (4.2) we obtain

$$c_1 = \frac{91}{34}, \qquad c_2 = -\frac{247}{34}, \qquad c_3 = -\frac{26}{17}.$$

Hence

$$\vartheta(\tau, F) = E(\tau, F) + \frac{91}{34}\vartheta(\tau, \varphi_{11}, Q_4) - \frac{247}{34}\vartheta(\tau, \varphi_{12}, Q_4) - \frac{26}{17}\vartheta(\tau, \varphi_{13}, Q_4).$$

Equating the coefficients of z^n on the both sides of this identity we deduce the following result.

Theorem 2. *The number of representations of the positive integer n by the quadratic form F is given by*

$$r(n, F) = \frac{24}{17}\sigma_3^*(n) + \frac{91}{34}\Big(\sum_{Q_4=n} x_1^2 - \frac{4}{13}n\Big)$$

$$-\frac{247}{34}\Big(\sum_{Q_4=n} x_1x_2 + \frac{1}{13}n\Big) - \frac{26}{17}\Big(\sum_{Q_4=n} x_1x_3\Big).$$

where

$$\sigma_3^*(n) = \begin{cases} (\sigma_3(n)) & \text{if} \quad 13 \nmid n, \\ \sigma_3(n) + 169\sigma_3(\frac{n}{13}) & \text{if} \quad 13|n. \end{cases}$$

This paper dedicated to my teacher Professor Giorgi Lomadze on the occasion of his 100th birthday.

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